

# EXPLORING PATTERNS OF KNOWLEDGE PRODUCTION

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MAY 2009

**Definition 1** (Knowledge). The term ‘knowledge’ is here used broadly to signify all forms of information production including those involved in technological innovation, cultural creativity and academic advance.

## 1. INTRODUCTION

Today, thanks to rapid advances in IT, we have available substantial datasets pertaining both to the *extent* **and** the *structure* of knowledge production across disciplines, space and time. Especially recent is the availability of good ‘structural’ data – that is data on the linkages and relationships of different pieces of knowledge, for example as provided by citation information. This new material allows us to explore the “patterns of knowledge production” in deeper and richer ways than ever previously possible and often using entirely new methods.

For example, it has long been accepted that innovation and creativity are *cumulative* processes, in which new ideas build upon old. However, other than anecdotal and case-study material provided by historians of ideas and sociologists of science there has been little data with which to study this issue – and almost none of a comprehensive kind that would make possible a systematic examination. However, the recent availability of comprehensive databases containing ‘citation’ information have allowed us to begin really examining the extent to which new work builds upon

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old – be it a new technology as represented by a patent or a new idea in academia as represented by a paper, builds upon old.

Similar opportunities present themselves in relation to identifying the creation of new fields of research or technology, and tracing their evolution over time. Here the existence of extensive "structural information" as presented, for example, by citation databases, enables new *systematic* approaches – for example, can new fields be identified (or perhaps defined) as points in 'knowledge space' far away from the existing loci of effort; or, alternatively, by the nature of its connections to the existing body of work.

Structural information of this kind can also be used in charting other changes in the life-cycle of knowledge creation. For example, to offer a specific conjecture, a field entering decline, though still exhibiting a similar level of output (papers etc) and even citations to a field in rude health, may display a citation structure which is markedly different – for example, more clustered within the field itself. Thus, by using this additional structural information we may be able to gain insights not available with simpler approaches.

At the same time, structure must also play a central role in any attempt to estimate knowledge related 'output' measures. This is of course not true for other forms of 'output', for example that of corn or steel, where we have relatively well-defined objective measures available: tonnes of such-and-such a quality.

But knowledge is different: the most obvious metrics, such as number of patents or papers produced, seem entirely inadequate: one particular innovation or paper may be 'worth' as much as a hundred or a thousand others. The issue here is that, compared to corn or steel, knowledge is extremely inhomogeneous, or put slightly differently, quality (or significance) differs very substantially across the individual pieces of knowledge (papers, patents etc). Thus, any serious attempt to measure the progress of knowledge must find some way to do this quality-adjustment and structural information seems essential to this.

## 2. WHAT SPECIFIC QUESTIONS MIGHT WE EXPLORE WITH SUCH DATASETS?

The following is a (non-exhaustive) list of the kinds of questions one might explore using these new datasets:

- (1) Can we use structure to infer information about quality of individual items?  
Clearly the answer is yes, for example by using a citation-based metric where a work's value is estimated based on its citation by others.
- (2) Can we then use this information together with more global structure of the production network to gain a better idea of total (quality-adjusted) output. This would allow one to chart progress, or the lack of it, over time?
- (3) Can we use structural information to investigate the life-cycle of fields? For example, can we see fields 'dying out' or the onset of diminishing returns? Can we see new fields coming into existence and their initial growth patterns?
- (4) What about productivity per capita and its variation across the population? It is likely that one would need to focus here within a discipline as it would be difficult to directly compare across disciplines, at least when using quality adjusted productivity.
- (5) Do the structures of knowledge production vary over time and across disciplines and does this have implications for their productivity? Can we compare the structure of evolution in technology or economics with that in 'natural' evolution and, if not, what are the primary differences?
- (6) How do other (observable) attributes related to the producers of knowledge (their collaboration with others, their geographical location) affect the structures we observe and the associated outcomes (output, productivity) already discussed above?
- (7) Do different policies (for example openness vs. closedness – weak vs. strong IP) have implications for the structure of production and hence for output and productivity?

- (8) Is knowledge production (in a particular area) ergodic or path-dependent?  
 Crudely: do we always end up in the same place or do small shocks have large long-term effects?

### 3. FRAMEWORK AND METHODOLOGY

**3.1. Framework.** We have a set of standardized ‘items’ of knowledge that we shall terms ‘works’: patents, papers, books, films etc. For each such ‘item’ we will have information on some set of attributes, such as:

- (1) Classificatory: e.g. keywords, subject classification etc
- (2) Temporal: e.g. when it was produced
- (3) Relational: e.g. citation data, or, more loosely, the set of other articles published in that journal
- (4) Miscellaneous: number of claims in the patent, length of book, journal in which article was published

Our aim is to utilize these sorts of information in order to answer our questions.

**3.2. Approach.** At the general level there are 3 basic methodological approaches we can take given the data:

**3.2.1. Vector Spaces.** Define vectors for each work by converting attributes into dimensions. For example, suppose we have a controlled vocabulary containing  $N$  terms used for classifying works, e.g. subject categories or keywords for patents or papers. Then, associated to each work  $i$ , we can define a length  $N$  vector  $x_i$  as the vector with 1 in column  $j$  if and only if the paper is classified with the  $j$ th entry in the controlled vocabulary.

Alternatively, if relational data is available, we can create a vector for each work by taking the dimensions to be the other works. That is, if there are  $N$  works in total, then work  $i$  is represented by a vector in an  $N$ -dimensional space with a 1 in position  $j$  if and only if it links to work  $j$ . (This equivalent to the  $i$ th row in the

standard adjacency matrix for the directed graph associated to this citation network – see below).

The beauty of converting works into vectors is that there is a *very* large existing literature on how to do classification, clustering and analysis of objects distributed in vector spaces.

3.2.2. *Graphs.* Where relational information exists it is natural to convert works into nodes and define edges between the nodes depending on whether the corresponding works are linked (where the relation is directed, for example in the case of citation, this will give a directed graph). The result of this will be a presentation of the set of works as a Graph. Standard graph-theoretic techniques can then be used.

3.2.3. *Temporality and Evolution.* If a temporal aspect is present it is natural to include this in the analysis. In the first case, this may simply involve stratifying by time when applying either of the two previous approaches (for example, only considering those works from a particular year). However, temporality is more than a simple filter, in particular, many of the processes we wish to examine, such as the birth and development of fields, are temporal in nature. Furthermore, in examining changes over time there are specific techniques and approaches that can be used.

## 4. EXAMPLES

4.1. **Introduction.** The data for most of the examples in this section come from Hall et al. (2001). Associated code can be obtained from the author (written using Python, SciPy and the NetworkX packages). The NBER dataset contains all patents issued between 1963 and 1999 (approx 2.3m) and a complete set of citations on all patents 1975-1999 (over 16m). Thus, we are working with a fairly sizable dataset and one which therefore poses some significant technical challenges from a computational and performance perspective.

4.2. **Example 1: The Degree Distribution.** A huge amount of intellectual energy has been exerted examining the citation distribution of different knowledge

‘networks’ going back to the pioneering work of Price (1965) and even earlier. In graph theoretic terms this would be the in-degree distribution – with the out-degree distribution being the distribution of ‘cites’. This distribution, and its properties, is considered important because citations are taken to represent, albeit very crudely, some measure of importance and therefore quality.

More recently the exact form of the degree distribution, both here and in other areas where graph theoretic models are suitable, has been the subject of renewed interest because of the possibility that certain features may be common across a set of diverse areas (itself an indicator that there may be some theoreticably modelable common process).<sup>1</sup>

The full citation distribution for the complete NBER dataset is shown in figure 1 in both log-log and semilog forms. On the crude chi-by-eye approach it looks like a power law is a good fit for the middle portion of the distribution (papers cited between 10 and  $\approx 300$  times) but is less good outside of these regions.<sup>2</sup>

One problem here of course, is that we have a truncated set of data (no patents before 1963) and citations only from after 1975. Combined with the major increase in the number of patents issued per year this may be having a significant impact on the resulting distribution. Figure 2 attempts to correct this by limiting to patents in the period 1975-1994.

Apart from the truncation issue (which may occur at either end of the distribution), there is also the question as to mixing distributions from different types of patents – pharmaceutical patents may have a different degree distribution. We therefore also calculated the citation distribution for individual subcategories. Figure 3 shows the degree distribution (log-log and semi-log) for subcategory 13 (the

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<sup>1</sup>More specifically, the existence of a power-law form for the degree distribution.

<sup>2</sup>As discussed in Clauset et al. (2007) ‘by-eye’, or even basic fit-tests, of power-laws are not very reliable. Instead a test should be conducted based on the Kolmogorov-Smirnov statistic.

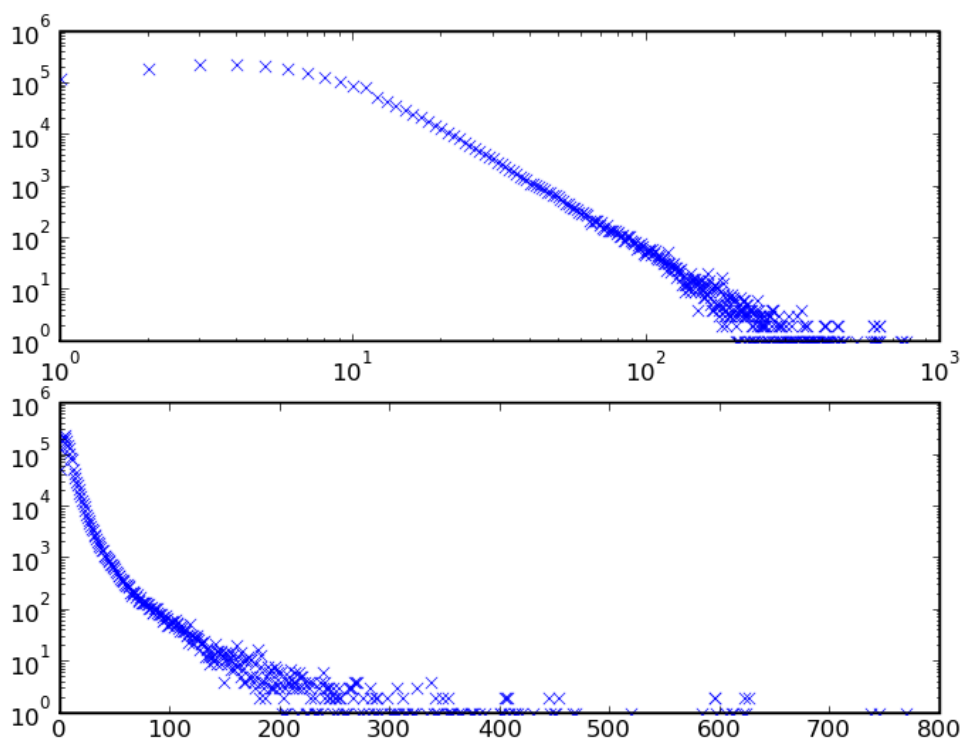


FIGURE 1. Citation Distribution for all Patents 1963-1999

smallest subcategory). While obviously different in scale, the distribution is reasonably similar, in particular in indicating that a power-law would seem a reasonable fit for a large middle portion of the distribution.

#### 4.3. Example 2: Using Citation Data to Relate Technological Categories.

Patents are classified by the patent office into (several hundred) classes. These in turn have been converted into a set of just over 30 technological subcategories by the creators of the NBER patent dataset.

In this first example, the basic ‘items’ rather than being the patents themselves were these technological subcategories. We constructed a weighted adjacency graph matrix to represent the flow of citations from one category to another in a given year (so  $A_{ij}$  = total number of citations from category  $i$  to category  $j$  in the relevant period).

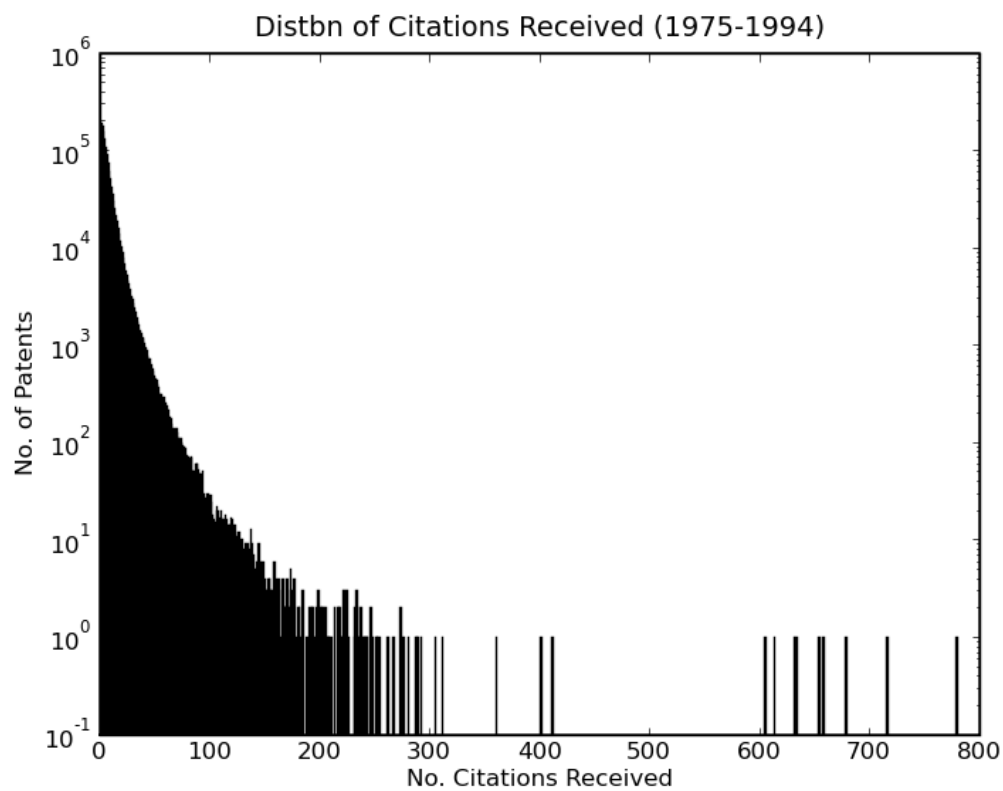


FIGURE 2. Citation Distribution for all Patents 1975-1994

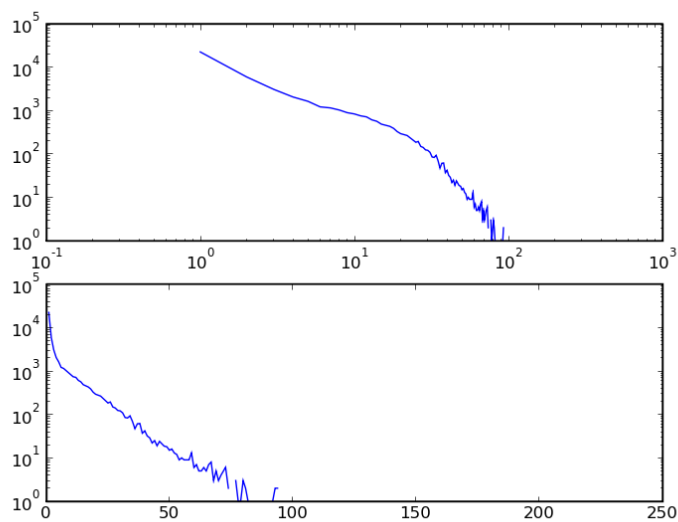


FIGURE 3. Citation Distribution for Subcategory 13



One can also see natural bridging groups, for example various (high-tech) mechanical and measuring categories in the middle-to-top left which connect to both the computer group and the chemical group.

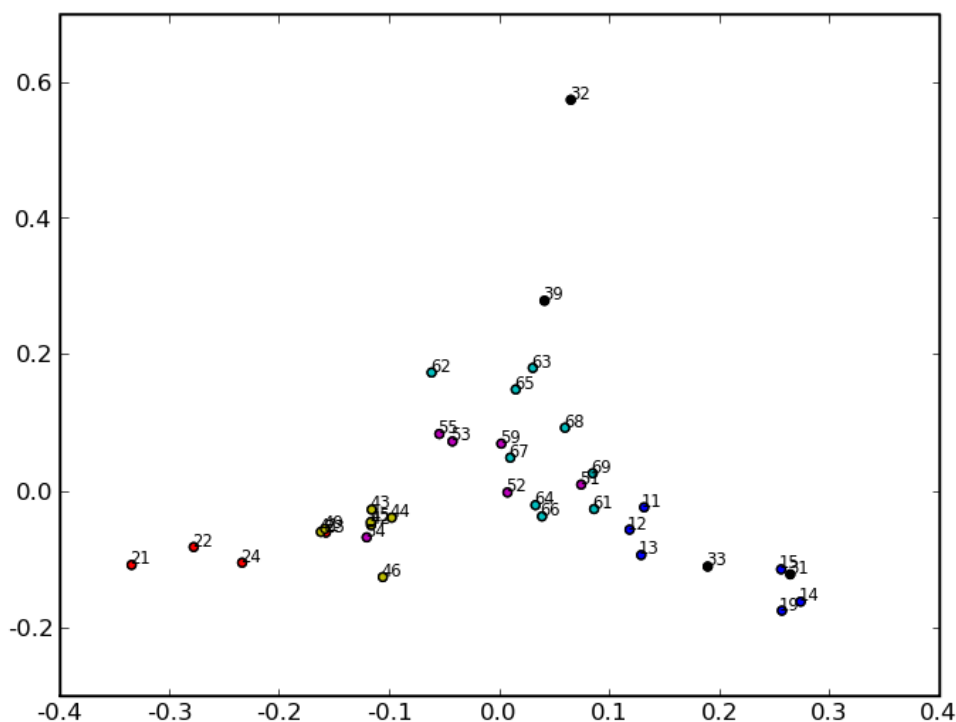


FIGURE 5. Relationship of Technological Subcategories Visualized Using PCA. Subcategories from the same main category have the same first digit and have been shaded the same colour. Table 1 gives a full listing of subcategories, their full names and associated main category.

An alternative to the graph theoretic approach Alternatively one can conduct a PCA analysis of the basic weighted adjacency matrix interpreting the flows as vectors in the standard way described above. Doing this results in figure 5. As we would hope, subcategories that share the same main category are placed close together by our PCA analysis of the citation flows.<sup>5</sup>

<sup>5</sup>Subcategories were formed by pulling together similar patent classes. Patent classes in turn are, in theory, there to classify similar material.

However, there is some notable variation. For example, some categories subcategories are much more tightly grouped than others – compare the 40s with the 30s. There are also some noticeable outliers: 32 (Surgery and Med. Instr.) and, to a lesser, extent, 39 (Misc Drugs and Chemicals). Some main categories show a clear division into two groups, such as the 1x grouping (Chemical) which splits crudely as ‘applications’ (11,12,13) and ‘pure chemicals’ (14,15,19). There are also some clear misclassifications, for example Optics (54) is in the Mechanical (5x) category but looks like it should be in the Electrical and Electronic (4x) category, similarly Computer Peripherals (23) looks like it should be in Electrical and Electronics (4x) rather than in Computer and Communications. Overall, the groupings one would impose via simple proximity via the PCA would look rather different from the actual category groupings we have: based on a crude by-eye approach, one would create 4-5 categories, one at bottom-left around 2x, one at mid-bottom-left around 4x, one bottom-right around 1x+31, one at top around 32, and a large amorphous grouping in the centre (perhaps split into 2).

**4.4. Example 3: Patents in Space.** This example uses a vector-based approach in which our ‘items’ are the patents from the NBER patent file and the vectors are based on the subcategory classifications given to patents. So with  $N = 36$  subcategories each patent is mapped to a vector  $x$  in an  $N = 36$  dimensional vector space. We term these vectors ‘subject’ vectors as they relate to the subject matter of the patent.

Each patent is classified to a single subcategory so under the simplest mapping each patent would be represented by a 1 in a single column. This would not be very interesting! Thus instead, motivated by the implied relationship involved in citation, we construct the vector for patent  $i$  using the subcategories of the patents that are cited by  $i$ .<sup>6</sup> To allow for the fact that patents vary substantially in the number of cites they make we normalized each vector by dividing by the total number of cites

<sup>6</sup>We could symmetrically have also used the categories of patents citing  $i$ . However, given the greater inequality in citation than in citing we preferred to focus on citing alone.

<b>Id</b>	<b>Name</b>	<b>Main Category</b>	<b>No. Patents</b>
11	Agriculture,Food,Textiles	Chemical	25624
12	Coating	Chemical	44366
13	Gas	Chemical	14331
14	Organic Compounds	Chemical	124981
15	Resins	Chemical	100725
19	Miscellaneous-chemical	Chemical	296907
21	Communications	Computers & Communications	122981
22	Computer Hardware & Software	Computers & Communications	91614
23	Computer Peripherals	Computers & Communications	24282
24	Information Storage	Computers & Communications	51460
31	Drugs	Drugs & Medical	84824
32	Surgery & Med Inst.	Drugs & Medical	70573
33	Biotechnology	Drugs & Medical	32170
39	Miscellaneous-Drgs&Med	Drugs & Medical	16632
41	Electrical Devices	Electrical & Electronic	99950
42	Electrical Lighting	Electrical & Electronic	46950
43	Measuring & Testing	Electrical & Electronic	84098
44	Nuclear & X-rays	Electrical & Electronic	42880
45	Power Systems	Electrical & Electronic	103534
46	Semiconductor Devices	Electrical & Electronic	52603
49	Miscellaneous-Elec	Electrical & Electronic	69726
51	Mat. Proc & Handling	Mechanical	167725
52	Metal Working	Mechanical	94679
53	Motors & Engines + Parts	Mechanical	109459
54	Optics	Mechanical	64848
55	Transportation	Mechanical	88856
59	Miscellaneous-Mechanical	Mechanical	155811
61	Agriculture,Husbandry,Food	Others	63994
62	Amusement Devices	Others	29619
63	Apparel & Textile	Others	55158
64	Earth Working & Wells	Others	43822
65	Furniture,House Fixtures	Others	61256
66	Heating	Others	40733
67	Pipes & Joints	Others	27151
68	Receptacles	Others	63173
69	Miscellaneous-Others	Others	256427

TABLE 1. List of Subcategories

that patent made. Following this we performed PCA. Figure 6 shows the results when applied to the smallest subcategory which contains approximately 14k patents (subcategory 13).

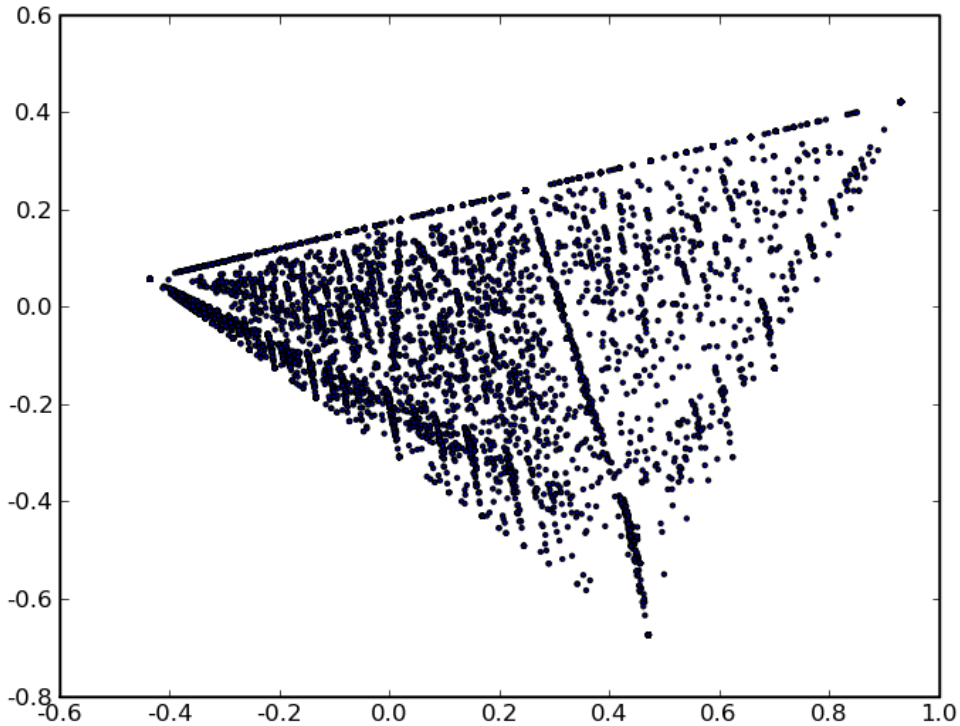


FIGURE 6. 2-d PCA for subcategory 13 subject vectors

As expected, normalization has led to a basic simplex arrangement (in the full 36d space all vectors  $x$  satisfy  $x_i \geq 0, \sum x_i = 1$ ). There is also some basic sub-structure visible but the density of points make it difficult to see much more than this.

In Figure 7 we plot two separate subcategories. Here, though slightly difficult to see because of the density of dots (it is worth ‘zooming in’), we can see a clear separation of the categories with the majority of the 13 (blue) in a subsimplex on the righthandside while subcat 12 (red) is spread across the simplex fairly evenly.<sup>7</sup> Of perhaps greater interest, zooming in, we can occasional blue dots dispersed away from the main concentration. These, then, are subcat 13 patents which are unusual in being more similar to a subcat 12 patent than a subcat 13. It would be possible

<sup>7</sup>The fact that subcat 12 determines the overall boundaries is not a coincidence here: subcat 12 is the larger category and hence will dominate the PCA to some extent (this could be corrected by randomly sampling to ensure all categories contributed an equal amount to the PCA).

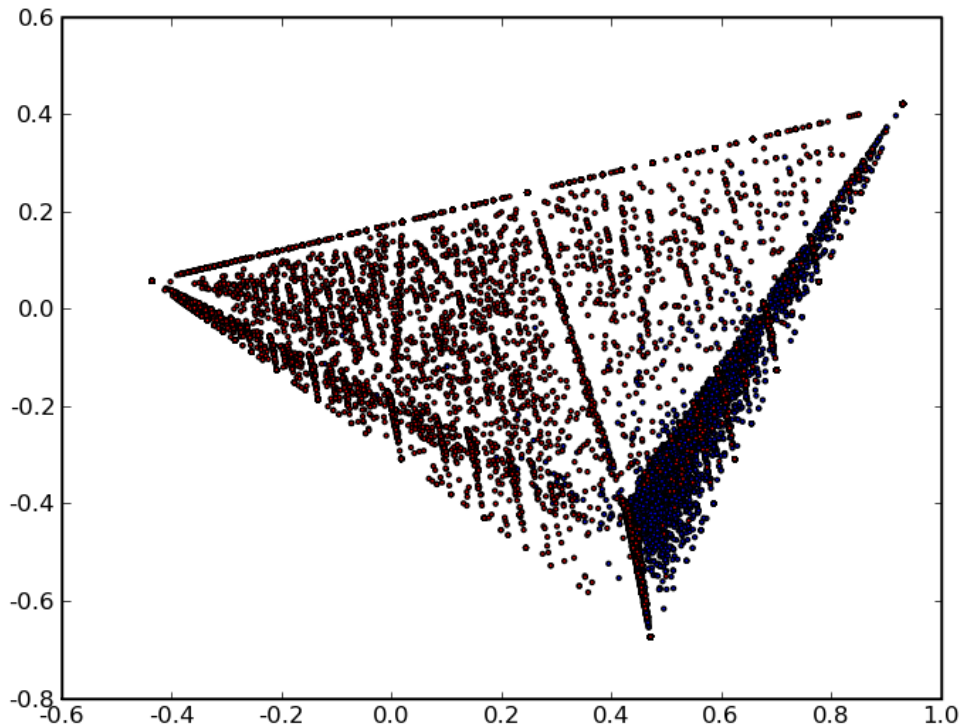


FIGURE 7. 2-d PCA for subcategory 12 (red) and 13 (blue) subject vectors

to algorithmically identify these patents and it would be interesting to see in what other ways they differ from their subcategory siblings (for example, do they have or make a higher share of citations?).

TODO: a) expand to more categories b) reduce number of items graphed by randomly selecting from a category (could also use correct bias in PCA layout due to different sized categories by equalizing number from each category – at least in constructing the PCA transformation ...) c) find algorithm for doing automated cluster identification

**4.5. Example 4: Graphing the Citation Network.** Another suggestive visualization is that provided by graphing the network of citations. Unfortunately, most layout algorithms are superlinear in network size and we are therefore limited in the scale of what can be shown. Figure 8, shows the full citation network (undirected

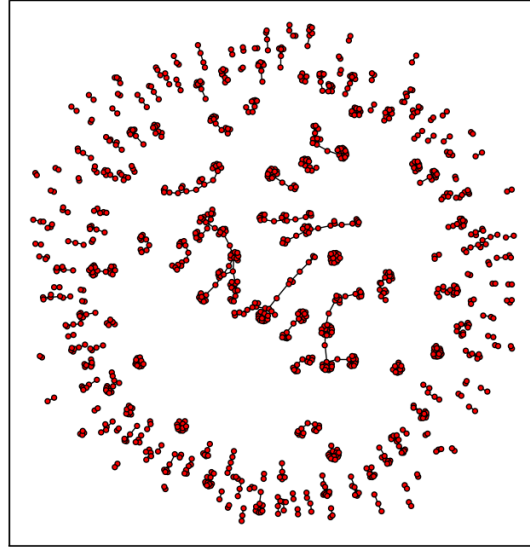


FIGURE 8. Spectral Visualization of Network of Citations for 1000 patents from Subcategory 13

links) for a 1000 patents from subcategory 13. Even with just a 1000 nodes the algorithm took several hours to complete on a modern laptop (by contrast a 100 node version finished in a few seconds). What can we see even from this limited example?

First, as is expected, many patents are either entirely isolated or form a pair. In our figure these are arranged around the periphery. Moving inwards things become more interesting. First, we can see what look like several star-like clusters with several (2+) patents link to a single central node. We also start to see a few cases of groupings with a diameter greater than 1. Moving right into the centre we have examples of fairly sizable clusters with some marked chain-like structure (with occasional branching). Remembering that each node corresponds to a patent and each link to a cite these chains can be taken, crudely, to equate to innovation pathways or trajectories (Verspagen (2005)).

#### 4.6. Further Suggestions.

- Citation networks as branching processes

- Evolution over time using dirichlet diffusion trees
- Classifying outliers and examining their properties
- Computing centrality (e.g. a primary eigenvector) for the citation network and correlating that with other standard features of item value (in degree, etc)
- Identifying items (e.g. patents) that fill ‘structural holes’

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